Comment on 'Comment on the Longitudinal Magnetic Field of Circularly Polarized Electromagnetic Waves' by E. Comay [Chemical Physics Letters 261 (1996) 601-604]*

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It is proved that necessary corrections in the Evans-Vigier modified electrodynamics invalidate the arguments given by E. Comay [CPL **261** (1996) 601] against this model. Moreover, from the conceptual viewpoint Evans/Comay discussions in several journals contributed very little to the modern electromagnetic theory due to many confusions of the both.

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Recently, the model proposed by M. Evans and J.-P. Vigier [1] was the object of the strong critics [2–4]. I cannot consider the replies of M. Evans *et al.* [5] as sufficient ones. In fact, they contributed additional confusions and misunderstandings to the discussion.[1] This discussion inspired me to express my own opinion on the problem of the longitudinal modes of the electromagnetic field, see, e. g., refs. [8,9] and the present paper is the continuation of my efforts to consider the problem *rigorously.* I have to mention that I disagree with both E. Comay and M. Evans *et al.*

First of all, one should repeat briefly what the authors of the cited works claimed. In ref. [1] the longitudinal "magnetic field" (an axial vector) $\mathbf{B}_\Pi \sim \mathbf{E}\times\mathbf{E}^*$ was ascribed to a circularly polarized electromagnetic wave. Moreover, in the subsequent papers and books the **B**- cyclic relations

$$\mathbf{B}^{(1)}\times\mathbf{B}^{(2)} = iB^{(0)}\mathbf{B}^{(3)\,*} \quad \text{(et cyclic)} \tag{1}$$

were derived. The $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)} = \mathbf{B}^{(1)\,*}$ are the accustomed transverse modes of the circularly polarized electromagnetic wave (see refs. [1,2] for detailed explanation of the notation):

$$\mathbf{B}^{(1)} = \frac{B^{(0)}}{\sqrt{2}}\begin{pmatrix} i\\ 1\\ 0\end{pmatrix} e^{i\phi} \quad , \quad \mathbf{B}^{(2)} = \frac{B^{(0)}}{\sqrt{2}}\begin{pmatrix} -i\\ 1\\ 0\end{pmatrix} e^{-i\phi} \quad . \tag{2}$$

Thus, the longitudinal phaseless component

$$\mathbf{B}^{(3)} = B^{(0)}\begin{pmatrix} 0\\ 0\\ 1\end{pmatrix} \tag{3}$$

of the "magnetic field" in the circular *complex* basis was defined there. This model has come across the strong critics. E. Comay recently argued that the model violates the relativistic covariance principle. See [8] for the discussion of whether this is so.[2] Furthermore, on the basis of the calculation of the line integral in the problem of rotating dipole [6, p.228 of the Russian edition][3] the author of [2] concluded that "the flux of the electric field **E** through the area *increases indefinitely* as time progresses. It follows that if the Maxwell equation

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[1]For instance, in the reply by M. Evans and S. Jeffers in FPL [5a] the authors 1) considered relations which are valid if the circular polarized radiation presents only; 2) in an attempt of a counterexample they considered another path of integration and, in fact, another type of radiation; and 3) contradicted the conclusions made in ref. [6] without sufficient explanations. So, in my opinion, their paper [5a] is irrelevant to the counterexamples presented by E. Comay. This was pointed out by G. Hunter [7].

[2]As opposed to the opinion of E. Comay [4, p. 252, 9th line from the bottom] the principle of relativistic covariance means that the physical laws expressed by equations preserve their form in *any* frame.

[3]In this problem the polarization is defined by the direction $\mathbf{n}\cdot\mathbf{d}$, $\mathbf{d}$ is the dipole moment [6, p.228

in the vacuum $\nabla \times \mathbf{B} = \partial \mathbf{E}/\partial t$ and (C)[4] hold then the modified electrodynamics leads to contradictions."

I agree. But, it is easy to show that if one corrects the erroneous statement of M. Evans that there cannot be any longitudinal components in the linear polarized electromagnetic wave, then the path integral over the segment SR (see the figure 2 in [2]) does not vanish and it gives the contribution to $\oint \mathbf{B} \cdot d\mathbf{l}$, which is equal in the magnitude and opposite in the sign to that of the path segment QP. The total path integral $\oint \mathbf{B} \cdot d\mathbf{l}$ is equal to zero, thus invalidating the arguments by E. Comay.

For quantum field theorists it is known that the change of the polarization state of massive particles can be made by the boost (and/or other non-unitary operations). On the other hand, it appears that for $j = 1$ states (relevant to the problem at hand) the change of polarization can be made by means of the change of the basis of the corresponding *complex* vector space, *i. e.* by the rotation. It is produced by an unitary matrix. If one describes the magnetic field as

$$\mathbf{B}^{\mathrm{circ.}} = \frac{B^{(0)}}{\sqrt{2}} \left[\begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix} e^{+i\phi} + \begin{pmatrix} -i \\ 1 \\ 0 \end{pmatrix} e^{-i\phi} \right] , \tag{4}$$

($\phi = \omega t - \mathbf{k} \cdot \mathbf{r}$) on using the unitary matrix

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} -i & 1 & 0 \\ i & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \tag{5}$$

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of the Russian edition], thus giving the circular, elliptical and linear polarizations when considering radiation emitted in various surface angles. It was claimed by M. Evans (see the reference in [2]) that the $\mathbf{B}^{(3)}$ is the property of the circular and, possibly, elliptic polarizations and is equal to zero (??) in the linear polarization (nevertheless, cf. [10]). So, the problem noted by E. Comay still may stands at the $\mathbf{B}^{(3)}$ theory, if one trusts the Evans claims and if one considers the longitudinal field as a part of an antisymmetric tensor of the second rank. Nevertheless, it is interesting to note that, apart from the presence of different polarizations, the energy flux is *not* isotropic in the particular example of Comay. It depends on the polar angle θ as argued by Landau [6, p.228 of the Russian edition] even in the case of the consideration of time-average flux over the period. All this may lead to further speculations on the nature of $\mathbf{B}^{(3)}$.

[4](C) stands for the Evans' claim that "the magnetic field $\mathbf{B}^{(3)}$ is not associated with any real electric field" which also may be doubted.

one can obtain the linear polarized (in the plane XY) radiation[5,6]

$$\mathbf{B}^{\rm lin.} = U\mathbf{B}^{\rm circ.} = B^{(0)}\left[\begin{pmatrix}1\\0\\0\end{pmatrix} e^{+i\phi} + \begin{pmatrix}0\\1\\0\end{pmatrix} e^{-i\phi}\right] . \tag{9}$$

For this case of the linear polarized radiation one has (instead of eq. (1))

$$B_x\mathbf{i}\times B_y\mathbf{j} = [B^{(0)}]^2\begin{pmatrix}0\\0\\1\end{pmatrix} = B^{(0)}B^{(0)}\mathbf{k}\,, \tag{10}$$

i. e., the similar relation to (1), but already without the phase factor $e^{i\pi/2}$. This conclusion is in the complete accordance with the Lakhtakia consideration [10a]: the Evans' 'magnetostatic' field $\mathbf{B}_\Pi$ (or, later, $\mathbf{B}^{(3)}$) "may be defined for other than circularly polarized plane waves".

These relations should be applied only in the *local* system, which is connected with the observation point and the wave vector. Otherwise, we come across big confusions. If one

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[5]If one wishes to see the real-valued magnetic fields instead of phasors here they are:

$$\mathbf{B}_x^{\rm circ.} = -\sqrt{2}B^{(0)}\sin\phi \quad , \mathbf{B}_y^{\rm circ.} = +\sqrt{2}B^{(0)}\cos\phi \quad . \tag{6}$$

or

$$\mathbf{B}_x^{\rm lin.} = +B^{(0)}\cos\phi \quad , \mathbf{B}_y^{\rm lin.} = +B^{(0)}\cos\phi \quad , \tag{7}$$

i. e., in the latter case one obtains the linear polarized radiation with the polarization angle equal to $\pi/4$ (defined by (7)). Of course, the given unitary matrix can be easily generalized to account for other polarization angles. Cf. with ref. [11, §7.2].

[6]The transformation of transverse components (2) with the matrix L used by G. Hunter is *not* generally unitary (cf. with formulas (19) in [7a]):

$$L \sim \begin{pmatrix}(A-B)\cos\alpha & -(A+B)\sin\alpha & 0\\ (A-B)\sin\alpha & (A+B)\cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}\,, \tag{8}$$

with α being the polar angle of the cylindrical system of coordinates. In the case of the linear polarization defined in such a way [7] one has $\mathbf{B}\times\mathbf{B}^* = 0$. This transformation may also change the normalization of the corresponding vectors which in the quantized case correspond to a particle and an anti-particle. The determinant of the transformation is, in general, *not* equal to the unit. While the determinant of our matrix is also not equal to the unit ($\det U = -i$), but the norm of the corresponding quantum states is still preserved (while this is not so for the corresponding real quantities). By the way, Landau in §67 did not work in terms of phasors; unfortunately, Dr. E. Comay did also not elaborate this point. So, we do not know, what the definition of linear polarized radiation does Landau imply in the problem of the rotating dipole, presented by G. Hunter or presented by me in this work? Nevertheless, cf. footnotes 3 and 5.

wishes to use the *global* system of coordinates for this problem $\mathbf{B} \sim \ddot{\mathbf{d}} \times \mathbf{n}$ is parallel to OZ in the case of the observation point in the plane XY; the vector cartesian components $\mathbf{B}$ are already angular dependent, what makes the calculations to be more difficult.

So, with necessary corrections the Evans-Vigier model can be considered as useful and uncontradictory.[7] In fact, the $\mathbf{B}$ cyclic relations repeat tautologically the relations between spin components (after taking into account the normalization), represent an interesting model, but hardly to be considered as a fundamental theory (at the present level of its development). Furthermore, one should note that the $\mathbf{B}^{(3)}$ theory is *not* the only candidate for the appear-to-be necessitated generalization of the Maxwell's formalism. As I am now aware the longitudinal components of electromagnetic radiation were considered by many authors in both XIX and XX centuries, e. g., refs. [9,15]. So, the common belief in the impossibility of existence of the longitudinal electromagnetic-type interactions appears to me to be the result of the greatest and uncomprehensible mistake in the history of the XX century science. In my opinion, the most intriguing and promising theory is the Weinberg $2(2j+1)$ component theory [16,17], which also represents the modified theory of electromagnetism [18] and [20a]. The Weinberg theory was shown to be related to the problem of the so-called Kalb-Ramond field [19] (as well as the Evans-Vigier model).

In relation with all the above-mentioned I demonstrated in my recent works (and this was let to know to Dr. Comay in 1995-1996) that:

- The 3-vector $\mathbf{B}^{(3)}$ (which is defined by (1)) may *not* be the entry of the antisymmetric tensor field [8]; it is *not* the $B_z \equiv F^{21}$ component but the entry of some 4-vector[8] provided that the Evans' definitions for circularly polarized radiation are used.[9] Lorentz transformation rules for $(B^{(0)}, \mathbf{B}^{(3)})$ are the following:

$$B^{(0)\prime} = \gamma(B^{(0)} - \boldsymbol{\beta}\cdot\mathbf{B}^{(3)}) \quad , \tag{11a}$$

$$\mathbf{B}^{(3)\prime} = \mathbf{B}^{(3)} + \frac{\gamma-1}{\beta^2}(\boldsymbol{\beta}\cdot\mathbf{B}^{(3)})\boldsymbol{\beta} - \gamma\boldsymbol{\beta}B^{(0)} \quad , \tag{11b}$$

  with $\boldsymbol{\beta} = \mathbf{v}/c$, $\beta = |\boldsymbol{\beta}| = \tanh\phi$, $\gamma = \frac{1}{\sqrt{1-\boldsymbol{\beta}^2}} = \cosh\phi$, and ϕ is the parameter of the Lorentz boost.

- Due to the previous item *there are no any reasons* that the quantity which is *not* a part of the antisymmetric tensor field $F^{\mu\nu}$ satisfies the Maxwell's vacuum equations $\partial_\mu F^{\mu\nu} = 0$, $\mu = 0, 1, 2, 3$.

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[7]See, nevertheless, the experimental controversy in refs. [12–14].

[8]This is obvious even from the fact that $\mathbf{B}^{(3)}$ is a 3-vector (can possess three components, in the general case) but B_z is a number, the entry of $F^{\mu\nu}$, the electromagnetic tensor. In the paper [8] we used the instant form of dynamics. It would be interesting to repeat the calculations in the light-front form of relativistic dynamics [21].

[9]It would be still interesting to produce complete investigation of the transformation properties of the cross products of transverse modes in the case of various definitions of polarization states.

- In [18] the Weinberg-Tucker-Hammer equation [16a] and [22]

$$\left[\gamma_{\alpha\beta}p_\alpha p_\beta + p_\alpha p_\alpha + 2m^2\right]\Psi(x^\mu) = 0 \qquad (12)$$

($p_\alpha = -i\partial_\alpha$ and the euclidean metric being used) was considered on using the interpretation of the Weinberg $j = 1$ field functions as $\Psi(x^\mu) = \mathrm{column}(\chi \quad \varphi)$, $\chi = \mathbf{E} + i\mathbf{B}$, $\varphi = \mathbf{E} - i\mathbf{B}$. As a result we arrive at the set of equations

$$\begin{aligned}&\left[E^2 - \mathbf{p}^2\right]_{ij}(\mathbf{E}^j + i\mathbf{B}^j)^\parallel - m^2(\mathbf{E}^i - i\mathbf{B}^i)^\parallel + \\ &+\left[E^2 + \mathbf{p}^2 - 2E(\mathbf{J}\cdot\mathbf{p})\right]_{ij}(\mathbf{E}^j + i\mathbf{B}^j)^\perp - m^2(\mathbf{E}^i - i\mathbf{B}^i)^\perp = 0 \quad ,\end{aligned} \qquad (13)$$

and

$$\begin{aligned}&\left[E^2 - \mathbf{p}^2\right]_{ij}(\mathbf{E}^j - i\mathbf{B}^j)^\parallel - m^2(\mathbf{E}^i + i\mathbf{B}^i)^\parallel + \\ &+\left[E^2 + \mathbf{p}^2 + 2E(\mathbf{J}\cdot\mathbf{p})\right]_{ij}(\mathbf{E}^j - i\mathbf{B}^j)^\perp - m^2(\mathbf{E}^i + i\mathbf{B}^i)^\perp = 0 \quad .\end{aligned} \qquad (14)$$

One can see that in the classical field theory antisymmetric tensor fields are the fields with both transverse and longitudinal components in the massless limit. The longitudinal parts of the above equations do not contain the terms as $(\mathbf{J}\cdot\mathbf{p})$ provided that the longitudinal modes are associated with the plane waves too. This can be easily seen on choosing the spin basis where $(J^i)_{jk} = -i\epsilon^{ijk}$ and on using the definition of the longitudinal modes, $\mathbf{p}\times(\mathbf{E}\pm i\mathbf{B})^\parallel \equiv 0$. So, the Weinberg-Tucker-Hammer equations for antisymmetric tensor fields (which are deduced on the basis of the general principles for deriving relativistic equations) may describe the longitudinal components with non-zero energy.

- If one considers the Maxwell's equations as the definitions for currents and charges one arrives at the additional equations [20a]:

$$\frac{\partial \mathbf{J}_e}{\partial t} + \mathrm{grad}\rho_e = m^2\mathbf{E} \quad , \quad \mathrm{curl}\mathbf{J}_m = 0\,, \qquad (15a)$$

$$\frac{\partial \mathbf{J}_m}{\partial t} + \mathrm{grad}\rho_m = 0 \quad , \quad \mathrm{curl}\mathbf{J}_e = -m^2\mathbf{B}\,, \qquad (15b)$$

$c = \hbar = 1$ and the indices e, m denotes electric and magnetic parts respectively. They might be relevant to the old Einstein idea of the dequantization of the charge and invoke immediately the additional concept of the scalar chi-functions of boundary and initial conditions. The massless limit is easily found from these formulas.

- In the recent paper [20d] we considered the general case of $2(2j+1)$ component field functions and 4-vector potential in the instant form of relativistic dynamics (cf. with [23]). The cross products of magnetic fields of different spin states in the momentum representation (such as $\mathbf{B}^{(+)}(\mathbf{p},\sigma)\times\mathbf{B}^{(-)}(\mathbf{p},\sigma')$) may *not* be equal to zero and may be expressed by the "time-like" potential and/or the gauge part of 3-potentials for different spin states (also in the momentum representation):

$$\mathbf{B}^{(+)}(\mathbf{p},+1)\times\mathbf{B}^{(-)}(\mathbf{p},+1) = -\frac{iN^2}{4m^2}p_3\begin{pmatrix}p_1\\ p_2\\ p_3\end{pmatrix} = -\mathbf{B}^{(+)}(\mathbf{p},-1)\times\mathbf{B}^{(-)}(\mathbf{p},-1)\ , \quad (16a)$$

$$\mathbf{B}^{(+)}(\mathbf{p},+1)\times\mathbf{B}^{(-)}(\mathbf{p},0) = -\frac{iN^2}{4m^2}\frac{p_r}{\sqrt{2}}\begin{pmatrix}p_1\\ p_2\\ p_3\end{pmatrix} = +\mathbf{B}^{(+)}(\mathbf{p},0)\times\mathbf{B}^{(-)}(\mathbf{p},-1)\ , \quad (16b)$$

$$\mathbf{B}^{(+)}(\mathbf{p},-1)\times\mathbf{B}^{(-)}(\mathbf{p},0) = -\frac{iN^2}{4m^2}\frac{p_l}{\sqrt{2}}\begin{pmatrix}p_1\\ p_2\\ p_3\end{pmatrix} = +\mathbf{B}^{(+)}(\mathbf{p},0)\times\mathbf{B}^{(-)}(\mathbf{p},+1)\ . \quad (16c)$$

N is the normalization term; $p_{r,l} = p_1 \pm ip_2$. Other cross products are equal to zero. Cf. with the formulas (15a,15b,22) in [20d].

Concluding, on the basis of this my paper and previous ones I can state that the possible existence of longitudinal components of antisymmetric tensor field (and/or 4-potentials) does not contradict the principle of relativistic covariance (but can still be related to the action-at-a-distance concept and topological theories); the curl of longitudinal components may satisfy the Maxwell equation after the necessary modifications of the claims made by M. Evans *et al.*, but this is not too necessary, because one can consider $\mathbf{B}^{(3)}$ to be the longitudinal components of 4-vector potentials (and/or of the polarization vector [7]), which need not already to satisfy the Maxwell equations for strengths. Finally, we found *two* ways for the definition of the linear polarized radiation; this freedom is related to the unobservability of phasors — only real electric/magnetic fields are observable in the present-day classical electrodynamics.

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[1] M. W. Evans, *Physica* B**182** (1992) 227; ibid. 237; M. W. Evans and J.-P. Vigier, *The Enigmatic Photon.* Vols. 1-3 (Kluwer Academic, Dordrecht, 1994-96), the third volume with S. Jeffers and S. Roy.

[2] E. Comay, *Chem. Phys. Lett.* **261** (1996) 601.
[3] E. Comay, *Physica* B**222** (1996) 150.
[4] E. Comay, *Found. Phys. Lett.* **10** (1997) 245.
[5] M. W. Evans and S. Jeffers, *Found. Phys. Lett.* **9** (1996) 587; M. W. Evans, *Found. Phys. Lett.* **10** (1997) 255; ibid. (1997) 403.
[6] L. D. Landau and E. M. Lifshitz, *Teoriya polya.* (Nauka, Moscow, 1973) [English translation: *The Classical Theory of Fields.* (Pergamon, Oxford, 1975)].
[7] G. Hunter, *The* $\mathbf{B}^{(3)}$ *Field: an Assessment.* Preprint, July 1997; *The* $\mathbf{B}^{(3)}$ *Field Controversy.* Preprint, July 1997.
[8] V. V. Dvoeglazov, *Found. Phys. Lett.* **10** (1997) 383 (This paper presents itself a comment on the debates between E. Comay and M. Evans, ref. [4,5b] and it criticizes both authors).
[9] V. V. Dvoeglazov, in *The Enigmatic Photon.* Vol. IV, eds. M. W. Evans, G. Hunter, S. Roy and J.-P. Vigier (Kluwer Academic, Dordrecht, 1997), Chapter 12.
[10] A. Lakhtakia, *Physica* B**191** (1993) 362; D. M. Grimes, ibid **191** (1993) 367.
[11] J. D. Jackson, *Electrodinámica Clásica.* Spanish edition (Alhambra S. A., 1980).
[12] J. P. van der Ziel, P. S. Pershan and L. D. Malmstrom, *Phys Rev. Lett.* **15** (1965) 190; *Phys. Rev.* **143** (1966) 574.
[13] G. L. J. A. Rikken, *Optics Lett.* **20** (1995) 846; M. W. Evans, *Found. Phys. Lett.* **9** (1996) 61.
[14] M. Y. A. Raja *et al.*, *Appl. Phys. Lett.* **67** (1995) 2123; *Appl. Phys.* B**64** (1997) 79.
[15] A. E. Chubykalo and R. Smirnov-Rueda, *Mod. Phys. Lett.* A **12** (1997) 1.
[16] S. Weinberg, *Phys. Rev.* B**133** (1964) 1318; ibid B**134** (1964) 882; ibid **181** (1969) 1893.
[17] D. V. Ahluwalia and D. J. Ernst, *Mod. Phys. Lett.* A**7** (1992) 1967; D. V. Ahluwalia, M. B. Johnson and T. Goldman, *Phys. Lett.* B**316** (1993) 102.
[18] V. V. Dvoeglazov, *Helv. Phys. Acta* **70** (1997) 677, ibid. 686, ibid. 697.
[19] V. I. Ogievetskii and I. V. Polubarinov, *Sov. J. Nucl. Phys.* **4** (1967) 156; K. Hayashi, *Phys. Lett.* B**44** (1973) 497; M. Kalb and P. Ramond, *Phys. Rev.* D**9** (1974) 2273.
[20] V. V. Dvoeglazov, Preprints EFUAZ-FT hep-th/9410174, hep-th/9604148, hep-th/9611068, hep-th/9712036, hep-ph/9801287 (1994-98), submitted.
[21] P. A. M. Dirac, *Rev. Mod. Phys.* **21** (1949) 392.
[22] R. H. Tucker and C. L. Hammer, *Phys. Rev. D***3** (1971) 2448.
[23] D. V. Ahluwalia and D. J. Ernst, *Int. J. Mod. Phys. E***2** (1993) 397.