Superconductors as transducers and antennas for gravitational and electromagnetic radiation

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Abstract

Type II superconductors will be considered as macroscopic quantum gravitational antennas, which can simultaneously also be used as efficient transducers for converting electromagnetic radiation into gravitational radiation, and vice versa. A Meissner-like effect, in which the Lense-Thirring field associated with a gravity wave is expelled from the interior of the superconductor, is predicted. An analysis of a process of natural impedance matching in type II superconductors such as YBCO based on the Ginzburg-Landau theory yields an estimate of the transducer conversion efficiency of the order of unity upon reflection of the wave. Thus efficient emitters and receivers of gravitational radiation can be constructed at microwave frequencies. A simple, Hertz-like experiment using YBCO and 12 GHz microwaves is being performed to test these ideas. Results of this experiment will be reported elsewhere.

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1 Introduction

In 1966, DeWitt [1] considered the interaction of a superconductor with gravitational fields, in particular with the Lense-Thirring field. Starting from the general relativistic Lagrangian for a single electron with a charge e and a mass m, he derived in the limit of weak gravity and slow particles a nonrelativistic Hamiltonian for a single electron in the superconductor, which satisfied the minimal-coupling rule

$$\mathbf{p} \to \mathbf{p} - e\mathbf{A} - m\mathbf{h}, \tag{1}$$

where **p** is the canonical momentum, **A** is the usual vector potential, and **h** is a gauge-like vector potential formed from the three space-time components g_{i0} of the metric tensor viewed as an ordinary three-vector. Papini [2] then considered the possibility of the detection the quantum phase shift induced by **h** arising from the Lense-Thirring field generated by a nearby rotating massive body, by means of a superconducting interference device (or SQUID) using Josephson junctions. (For recent work along these lines, see [3].) In a series of papers in the early 1980s, Anandan and I considered the possibility of constructing antennas for time-varying Lense-Thirring fields, and thus for gravitational radiation, using Josephson junctions as transducers, in *neutral* superfluid helium analogs of the SQUID using an antenna geometry in the form of a figure 8 superfluid loop, and also an antenna bent into a the form of a baseball seam [4]. In 1985, Anandan [5] considered the possibility of using superconducting circuits as *detectors* for *astrophysical sources* of gravitational radiation, but did not mention the possibility of superconductors being efficient *emitters*, and thus, *laboratory sources* of gravity waves, as is considered here. In this paper, I shall show that the use of Josephson junctions, which are difficult to implement experimentally, is unnecessary, and that a superconductor, but not superfluid helium, should by itself be a *direct* transducer from electromagnetic to gravitational radiation upon reflection of the wave from a superconductor-vacuum interface, with good conversion efficiency. By reciprocity, this conversion process can be reversed, so that gravitational radiation can also be converted upon reflection into electromagnetic radiation from the same interface, with equal efficiency. The geometry of a superconducting slab-shaped antenna proposed here is much simpler than that of the earlier proposed antenna geometries. These developments open up the possibility of a Hertz-like experiment, in which the emission and the reception of gravitational radiation at microwave frequencies can be implemented by means of a pair of superconductors used as transducers. This simple experiment is presently being performed.

2 Calculation of a Meissner-like effect in the linear response of a superconductor to gravitational radiation

Consider a gravitational plane wave propagating along the z axis, which impinges at normal incidence upon the circular face of a superconductor in the form of a large circular slab of radius r_0 and of thickness d. Let the radius r_0 be much larger than the wavelength λ of the plane wave, so that one can neglect diffraction effects. For simplicity, let the superconductor be at a temperature of absolute zero, so that only quantum effects need to be considered. The calculation of the coupling energy of the superconductor in the simultaneous presence of both electromagnetic and gravitational fields starts from the general relativistic Lagrangian for a single particle of rest mass m and charge e

(i.e., an electron, but neglecting its spin)

$$L = -m(-g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu})^{1/2} + eA_{\mu}\dot{x}^{\mu}, \tag{2}$$

from which a minimal-coupling form of the nonrelativistic Hamiltonian for an electron in a superconductor, in the limit of *weak* gravitational fields and *low* velocities, has been derived by DeWitt [1]. Here, let us apply this minimal-coupling Hamiltonian to a *pair* of electrons, i.e., a Cooper pair in a spin zero state,

$$H = \frac{1}{2m_{2eff}}\left(\mathbf{p} - e_2\mathbf{A} - m_2\mathbf{h}\right)^2, \tag{3}$$

where $m_2 = 2m$ is the rest mass of the Cooper pair, m_{2eff} is its effective mass, $e_2 = 2e$ is its charge, $\mathbf{p}$ is its canonical momentum, $\mathbf{A}$ is the electromagnetic vector potential, and $\mathbf{h}$ is the gravitomagnetic vector potential, which is the gravitational analog of $\mathbf{A}$ in the case of weak gravity. The vector potential $\mathbf{h}$ is the three-velocity formed from the space-time components h_{i0} of the small deviations of the metric tensor $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ from flat spacetime (the metric tensor being given by $g_{\mu\nu}$, and the Minkowski tensor for flat spacetime being given by $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$). Thus $\mathbf{h}|_i \equiv h_{i0}c$. It is convenient for performing this calculation to choose the radiation gauge for both $\mathbf{A}$ and $\mathbf{h}$, so that

$$\nabla \cdot \mathbf{A} = \nabla \cdot \mathbf{h} = 0. \tag{4}$$

The coordinate system used here is the inertial frame which coincides with the freely-falling center of mass of the superconductor at the origin, where the observer is located, and is *not* the transverse-traceless gauge choice, where h_{i0} is chosen to be zero. The physical meaning of $\mathbf{h}$ is that it is the *negative* of the three-velocity field of a system of noninteracting, locally freely-falling classical test particles as seen by the observer. In Eq. (3), Cooper pairs are treated as if they were free particles inside the superconductor, and we have neglected for the moment their interactions with each other.

The electromagnetic vector potential $\mathbf{A}$ in the above minimal-coupling Hamiltonian gives rise to Aharonov-Bohm interference. In like manner, the gravitomagnetic vector potential $\mathbf{h}$ gives rise to a general relativistic twin paradox for rotating coordinate systems and for Lense-Thirring fields. Therefore $\mathbf{h}$ gives rise to Sagnac interference in both light and matter waves. The Sagnac effect has recently been observed in superfluid helium interferometers using Josephson junctions, and has been used to detect the Earth's rotation around its polar axis [6].

From the above Hamiltonian, we see that the minimal coupling rule in the quantum mechanics (QM) of Cooper pairs now becomes

$$\mathbf{p} \rightarrow \mathbf{p} - e_2\mathbf{A} - m_2\mathbf{h} \tag{5}$$

in the simultaneous presence of electromagnetic (EM) and weak general relativistic (GR) fields. This minimal-coupling rule has been experimentally tested in the static case of a uniformly rotating superconducting ring, since it predicts the existence of a London magnetic moment for the rotating superconductor, in which magnetic flux is generated through the center of the ring due to its rotational motion with respect to the local inertial frame at its center of mass. The proportionality constant of the London moment effect is given by the ratio of the $e_2\mathbf{A}$ and the $m_2\mathbf{h}$ terms, and thus by the charge-to-mass ratio e_2/m_2, where m_2 has been experimentally determined to be the *vacuum* value of the Cooper pair rest mass, apart from a small discrepancy of the order of ten parts per million, which has not yet been completely understood [7].

I propose that we can generalize the above time-independent minimal-coupling Hamiltonian to quasi-static time-varying situations as follows:

$$H = \frac{1}{2m_{2eff}} \left(\mathbf{p} - e_2\mathbf{A}(t) - m_2\mathbf{h}(t)\right)^2 , \tag{6}$$

where $\mathbf{A}(t)$ and $\mathbf{h}(t)$ are the vector potentials associated with low-frequency electromagnetic and gravitational radiation fields, for example. (This time-dependent Hamiltonian can also of course describe time-varying tidal fields and Lense-Thirring fields, as well as radiation fields.) Again, it is natural to choose the radiation gauge, Eq. (4), in the description of these time-varying fields. The physical meaning of $\mathbf{h}(t)$ is that it is the *negative* of the time-varying three-velocity field $\mathbf{v}_{test}(x, y, z, t)$ of a system of noninteracting, locally freely-falling classical test particles as seen by the observer in an inertial frame located at the center of mass of the superconductor. At first, we shall treat both $\mathbf{A}(t)$ and $\mathbf{h}(t)$ as classical fields, but shall treat the matter, i.e., the superconductor, quantum mechanically, in the standard semiclassical approximation. The time-dependent Hamiltonian given by Eq. (6) is, I stress, only a "guessed" form of the Hamiltonian, whose ultimate justification must be an experimental one. In case of the time-dependent vector potential $\mathbf{A}(t)$, there have already been many experiments which have justified this "guess," but there have been no experiments which have tested the new term involving $\mathbf{h}(t)$. However, one justification for this new term is that in the static limit, this "guessed" Hamiltonian goes over naturally to the static minimal-coupling form, which, as we have seen above, *has* been tested experimentally.

From Eq. (6), we see that the time-dependent generalization of the minimal-coupling rule in QM is

$$\mathbf{p} \to \mathbf{p} - e_2\mathbf{A}(t) - m_2\mathbf{h}(t). \tag{7}$$

It would be hard to believe that one is allowed to generalize $\mathbf{A}$ to $\mathbf{A}(t)$, but that somehow one is *not* allowed to generalize $\mathbf{h}$ to $\mathbf{h}(t)$ for *quasi-static* time-varying fields.

One important consequence that follows immediately from expanding the square in Eq. (6) is that there exists a cross-term [8]

$$H_{int} = \frac{1}{2m_{2eff}} \left\{2e_2m_2\mathbf{A}(t) \cdot \mathbf{h}(t)\right\} = \left(\frac{m_2}{m_{2eff}}\right) e_2\mathbf{A}(t) \cdot \mathbf{h}(t). \tag{8}$$

Note that Newton's constant G does not enter here. The physical meaning of this interaction Hamiltonian H_{int} is that there should exist a *direct* coupling between electromagnetic and gravitational radiation mediated by the superconductor that involves the charge e_2 as its coupling constant. Thus the strength of this coupling is electromagnetic, and not gravitational, in its character. Furthermore, the $\mathbf{A} \cdot \mathbf{h}$ form of H_{int} implies that there should exist a *linear and reciprocal* coupling between these two radiation fields. This implies the possibility that the superconductor can be used as a *transducer* between these two forms of radiation, which can, in principle, convert power from one form of radiation into the other, and vice versa, with equal efficiency.

We can see more clearly the physical significance of the interaction Hamiltonian H_{int} once we convert it into second quantized form and express it in terms of the creation and annihilation operators for the positive frequency parts of the two radiation fields, as in the theory of quantum optics, so that in the rotating-wave approximation

$$H_{int} \propto a^{\dagger} b + b^{\dagger} a \tag{9}$$

where the annihilation operator a and the creation operator $a^{\dagger}$ of the quantized excitations of the single classical mode of the plane-wave electromagnetic radiation field corresponding to the amplitude A_{+} (see Eq. (13)), obey the commutation relation $[a, a^{\dagger}] = 1$, and where the annihilation operator b and the creation operator $b^{\dagger}$ of the quantized excitations of the single classical mode of the plane-wave gravitational radiation field corresponding to the amplitude h_{+} (see Eq. (14)), obey the commutation relation $[b, b^{\dagger}] = 1$. (This represents a crude, first attempt at quantizing the gravitational field, which applies only in the case of weak gravity and slow velocities.) The first term $a^{\dagger} b$ then corresponds to the process in which a graviton is annihilated and a photon is created inside the superconductor, and similarly the second term $b^{\dagger} a$ corresponds to the reciprocal process, in which a photon is annihilated and a graviton is created inside the superconductor. Energy is conserved by both of these processes. Time-reversal symmetry, and hence reciprocity, is respected by this interaction Hamiltonian.

Let us now introduce the purely quantum concept of *wavefunction*, in conjunction with the quantum adiabatic theorem. To obtain the response of the superconductor, we must make explicit use of the fact that the ground state wavefunction of the system is unchanged (i.e., to use London's term, "rigid") during the quasi-static time variations of both $\mathbf{A}(t)$ and $\mathbf{h}(t)$. The condition for validity of the quantum adiabatic theorem here is that the frequency of the perturbations $\mathbf{A}(t)$ and $\mathbf{h}(t)$ must be low enough compared with the BCS gap frequency of the superconductor, so that no transitions are allowed out of the BCS ground state of the system into any of the excited states of the system. However, "low enough" can, in practice, still mean quite high frequencies, e.g., microwave frequencies in the case of high T_c superconductors, so that it becomes practical for the superconductor to become comparable in size to a microwave wavelength λ.

Using the quantum adiabatic theorem, one obtains in first-order perturbation theory the coupling energy $\Delta E_{int}^{(1)}$ of the superconductor in the simultaneous presence of both $\mathbf{A}(t)$ and $\mathbf{h}(t)$ fields, which is given by

$$\Delta E_{int}^{(1)} = \left(\frac{m_2}{m_{2eff}}\right) \langle\psi\left| e_2\mathbf{A}(t)\cdot\mathbf{h}(t)\right|\psi\rangle =$$

$$\left(\frac{m_2}{m_{2eff}}\right) \iiint dxdydz\ \psi^*(x,y,z)\mathbf{A}(x,y,z,t)\cdot\mathbf{h}(x,y,z,t)\psi(x,y,z) \tag{10}$$

where

$$\psi(x,y,z) = \left(N/\pi r_0^2 d\right)^{1/2} = \text{constant} \tag{11}$$

is the Cooper-pair condensate wavefunction or Ginzburg-Landau order parameter of a homogeneous superconductor [9], the normalization condition having been imposed that

$$\iiint dxdydz\ \psi^*(x,y,z)\psi(x,y,z) = N, \tag{12}$$

where N is the total number of Cooper pairs in the superconductor. Let us assume that both $\mathbf{A}(t)$ and $\mathbf{h}(t)$ have the same ("+") polarization of quadrupolar radiation [10], and that both plane waves impinge on the slab of superconductor at normal incidence upon its circular face; then in Cartesian coordinates,

$$\mathbf{A}(t) = (A_1(t), A_2(t), A_3(t)) = \frac{1}{2}(x,-y,0)A_+\cos(kz-\omega t) \tag{13}$$

$$\mathbf{h}(t) = (h_1(t), h_2(t), h_3(t)) = \frac{1}{2}(x,-y,0)h_+\cos(kz-\omega t). \tag{14}$$

One then finds that the time-averaged interaction or coupling energy in the rotating-wave approximation between the electromagnetic and gravitational radiation fields mediated by the superconductor is

$$\overline{\Delta E_{int}^{(1)}} = \frac{1}{16}\left(\frac{m_2}{m_{2eff}}\right) N e_2 A_+ h_+ r_0^2. \tag{15}$$

Note the presence of the factor N, which can be very large, since it can be on the order of Avogadro's number N_0.

The calculation for the above coupling energy $\overline{\Delta E_{int}^{(1)}}$ proceeds along the same lines as that for the Meissner effect of the superconductor, which is based on the diamagnetism term H_{dia} in the expansion of the *same* time-dependent minimal-coupling Hamiltonian, Eq. (6), given by

$$H_{dia} = \frac{1}{2m_{2eff}}\left\{e_2\mathbf{A}(t)\cdot e_2\mathbf{A}(t)\right\}. \tag{16}$$

This leads to an energy shift of the system, which, in first-order perturbation theory, again in the rotating-wave approximation, is given by

$$\overline{\Delta E_{dia}^{(1)}} = \frac{1}{32m_{2eff}} N e_2^2 A_+^2 r_0^2. \tag{17}$$

Again, note the presence of the factor N, which can be on the order of Avogadro's number N_0. We know from experiment that the size of this energy shift is sufficiently large to cause a complete expulsion of the magnetic field from the interior of the superconductor, i.e., a Meissner effect. Hence we expect to see a complete reflection of the electromagnetic wave from the interior of the superconductor, apart from a thin surface layer of the order of the London penetration depth. All forms of diamagnetism, including the Meissner effect, are purely quantum effects.

Similarly, there is a "gravitodiamagnetic" term H_{Gdia} in the expansion of the *same* minimal-coupling Hamiltonian given by

$$H_{Gdia} = \frac{1}{2m_{2eff}} \left\{ m_2 \mathbf{h}(t) \cdot m_2 \mathbf{h}(t) \right\}. \tag{18}$$

This leads to a gravitodiamagnetic energy shift of the system given in first-order perturbation theory in the rotating-wave approximation by

$$\overline{\Delta E_{Gdia}^{(1)}} = \frac{1}{32m_{2eff}} N m_2^2 h_+^2 r_0^2. \tag{19}$$

3 The impedance of free space for gravitational radiation

It is not enough merely to calculate the coupling energy arising from the interaction Hamiltonian given by Eq. (15). We must also compare how large this coupling energy is with respect to the free-field energies of the uncoupled problem, in particular, that of the gravitational radiation, in order to see how big an effect we expect to see in the gravitational sector. To this end, I introduce here the concept of *impedance matching*, both between the superconductor and free space in both forms of radiation, and also between the two kinds of waves inside the superconductor viewed as a transducer. The impedance matching problem determines the *efficiency of the power transfer* from the antenna to free space, and from one kind of wave to the other. I therefore also introduce here the concept of the impedance of free space Z_G for a gravitational plane wave, in analogy with the concept of the impedance of free space Z_0 for an electromagnetic plane wave (here SI units are more convenient to use than Gaussian cgs units)

$$Z_0 = \frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 \text{ ohms}, \tag{20}$$

where μ_0 is the magnetic permeability of free space, and ε_0 is the dielectric permittivity of free space.

The physical meaning of the "impedance of free space" in the electromagnetic case is that when a plane wave impinges on a large, but thin, resistive film at normal incidence, due to this film's ohmic losses, the wave can be substantially absorbed and converted into heat if the resistance per square element of this film is comparable to 377 ohms. In this case, we say that the electromagnetic plane wave has been approximately "impedance-matched" into the film.

If, however, the resistance of the thin film is much lower than 377 ohms per square, as is the case for a superconducting film, then the wave will be reflected by the film. In this case, we say that the wave has been "shorted out" by the superconducting film, and that therefore this film reflects electromagnetic radiation like a mirror.

By contrast, if the resistance of a normal metallic film is much larger than 377 ohms per square, then the film is essentially transparent to the wave. As a result, there will be almost perfect transmission.

The boundary value problem for Maxwell's equations coupled to a thin resistive film with a resistance per square element of $Z_0/2$, yields a unique solution that this is the condition for the *maximum* possible fractional absorption of the wave energy by the film, which is 50%, along with 25% of the wave energy being transmitted, and the remaining 25% being reflected (see Appendix A) [11]. Under such circumstances, we say that the film has been "*optimally* impedance-matched" to the film. This result is valid no matter how thin the "thin" film is.

The gravitomagnetic permeability μ_G of free space is [12][13]

$$\mu_G = \frac{16\pi G}{c^2} = 3.73 \times 10^{-26}\ \frac{\mathrm{m}}{\mathrm{kg}}, \tag{21}$$

i.e., μ_G is the coupling constant which couples the Lense-Thirring field to sources of mass current density, in the gravitational analog of Ampere's law for weak gravity. From this I find that

$$Z_G = \sqrt{\frac{\mu_G}{\varepsilon_G}} = \mu_G c = \frac{16\pi G}{c} = 1.12 \times 10^{-17}\ \frac{\mathrm{m}^2}{\mathrm{s}\cdot\mathrm{kg}}, \tag{22}$$

where the fact has been used that in GR both electromagnetic and gravitational radiation plane waves propagate at the same speed

$$c = \frac{1}{\sqrt{\varepsilon_G \mu_G}} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.00 \times 10^8\ \frac{\mathrm{m}}{\mathrm{s}}. \tag{23}$$

Therefore, the gravitoelectric permittivity ε_G of free space is

$$\varepsilon_G = \frac{1}{16\pi G} = 2.98 \times 10^8\ \frac{\mathrm{kg}^2}{\mathrm{N}\cdot\mathrm{m}^2}. \tag{24}$$

Note that Newton's constant G enters explicitly into the expression for Z_G (see Eq. (22)), and therefore that the impedance of free space for gravitational plane waves is an extremely small quantity. However, it is important to note that although the impedance Z_G is many orders of magnitude smaller than the corresponding quantity Z_0 for electromagnetic radiation, it is not strictly zero. Since nondissipative quantum fluids, such as superfluids and superconductors, can have strictly zero losses, they can behave like "short circuits" for gravitational radiation. Thus we expect that quantum fluids, in contrast to classical fluids, can behave like perfect mirrors for gravitational radiation. That Z_G is so small explains why it is so difficult to couple classical matter to gravitational radiation. It is therefore natural to consider using nondissipative quantum matter instead of dissipative classical matter for achieving an efficient coupling to gravity waves.

By analogy with the electromagnetic case, the physical meaning of the "impedance of free space" Z_G is that when a gravitational plane wave impinges on a large, but thin, viscous fluid film at normal incidence, due to this film's dissipative losses, the wave can be substantially absorbed and converted into heat, if the dissipation per square element of this film is comparable to Z_G. Again in this case, we say that the gravitational plane wave has been approximately "impedance-matched" into the film.

If, however, the dissipation of the thin film is much lower than Z_G, as is the case for nondissipative quantum fluids, such as a superconductor or a superfluid, then the wave will be reflected by the film. In this case, in analogy with the electromagnetic case, we say that the wave has been "shorted out" by the superconducting or superfluid film, and that therefore the film should reflect gravitational radiation like a mirror.

By contrast, if the dissipation of the film is much larger than Z_G, as is usually the case for classical matter, then the film is essentially transparent to the wave, and there will be almost perfect transmission.

The same boundary value problem obtains for the gravitational Maxwell-like equations coupled to a thin viscous fluid film with a dissipation per square element of $Z_G/2$, and yields the same unique solution that this is the condition for the *maximum* possible fractional absorption of the wave energy by the film, which is 50%, along with 25% of the wave energy being transmitted, and the remaining 25% being reflected (see Appendix A). Under such circumstances, we again say that the film has been "*optimally* impedance-matched" to the film. Again, this result is valid no matter how thin the "thin" film is.

When the superconductor is viewed as a transducer, the conversion from electromagnetic to gravitational wave energy, and vice versa, can be viewed as an $effective$ dissipation mechanism, where instead of being converted into heat, one form of wave energy is converted into the other form, whenever impedance matching is achieved within a thin layer inside the superconductor. As we shall see, this occurs naturally since the electromagnetic wave impedance is reduced exponentially in type II superconductors as the wave penetrates into the superconductor, so that a layer is automatically reached in its interior where the electromagnetic wave impedance is reduced to a level comparable to Z_G. Under such circumstances, which I shall call "natural impedance matching," we should expect efficient conversion from one form of wave energy to the other.

For obtaining Z_0, we recall that one starts from Maxwell's equations

$$\nabla \cdot \mathbf{D} = +\rho_e \tag{25}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{26}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{27}$$

$$\nabla \times \mathbf{H} = +\,\mathbf{j}_e + \frac{\partial \mathbf{D}}{\partial t}, \tag{28}$$

where ρ_e is the electrical free charge density (here, the charge density of Cooper pairs), and $\mathbf{j}_e$ is the electrical current density (due to Cooper pairs), $\mathbf{D}$ is the displacement field, $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic induction field, and $\mathbf{H}$ is the magnetic field intensity. The constitutive relations (assuming an isotropic medium) are

$$\mathbf{D} = \kappa_e \varepsilon_0 \mathbf{E} \tag{29}$$

$$\mathbf{B} = \kappa_m \mu_0 \mathbf{H} \tag{30}$$

$$\mathbf{j}_e = \sigma_e \mathbf{E}, \tag{31}$$

where κ_e is the dielectric constant of the medium, κ_m is its relative permeability, and σ_e is its electrical conductivity. We then convert Maxwell's equations into wave equations for free space in the usual way, and conclude that the speed of electromagnetic waves in free space is $c = (\varepsilon_0 \mu_0)^{-1/2}$, and that the impedance of free space is $Z_0 = (\mu_0/\varepsilon_0)^{1/2}$. The impedance-matching problem of a plane wave impinging on a thin, resistive film is solved by using standard boundary conditions in conjunction with the constitutive relation $\mathbf{j}_e = \sigma_e \mathbf{E}$ (see Appendix A).

Similarly, for weak gravity and slow matter, Maxwell-like equations have been derived from Einstein's field equations [13][14][15]. The gravitoelectric field $\mathbf{E}_G$, which is identical to the local acceleration due to gravity $\mathbf{g}$, is analogous to the electric field $\mathbf{E}$, and the gravitomagnetic field $\mathbf{B}_G$, which is identical to the Lense-Thirring field, is analogous to the magnetic field $\mathbf{B}$; they are related to the vector potential $\mathbf{h}$ in the radiation gauge as follows:

$$\mathbf{g} = -\frac{\partial \mathbf{h}}{\partial t} \text{ and } \mathbf{B}_G = \nabla \times \mathbf{h}, \tag{32}$$

which correspond to the electromagnetic relations

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \text{ and } \mathbf{B} = \nabla \times \mathbf{A}\ . \tag{33}$$

The physical meaning of $\mathbf{g}$ is that it is the three-acceleration of a local, freely-falling test particle induced by the gravitational radiation, as seen by an observer in a local inertial frame located at the center of mass of the superconductor. The local three-acceleration $\mathbf{g}$ is the local time derivative of the local three-velocity $-\mathbf{h}$ of this test particle, which is in a system of noninteracting, locally freely-falling, classical test particles with a velocity field $\mathbf{v}_{test}(x,y,z,t) = -\mathbf{h}(x,y,z,t)$ as seen by the observer (see Eq. (32a)). Similarly, the physical meaning of the gravitomagnetic field $\mathbf{B}_G$ is that it is the local angular velocity of an inertial frame centered on the same test particle, with respect to the observer's inertial frame, which is centered on the center-of-mass of the superconductor. Thus $\mathbf{B}_G$ is the Lense-Thirring field induced by the gravitational radiation.

The Maxwell-like equations for weak gravitational fields (upon setting the PPN ("Parametrized Post-Newton") parameters to be those of general relativity) are [14]

$$\boldsymbol{\nabla} \cdot \mathbf{D}_G = -\rho_G \tag{34}$$

$$\boldsymbol{\nabla} \times \mathbf{g} = -\frac{\partial \mathbf{B}_G}{\partial t} \tag{35}$$

$$\boldsymbol{\nabla} \cdot \mathbf{B}_G = 0 \tag{36}$$

$$\boldsymbol{\nabla} \times \mathbf{H}_G = -\,\mathbf{j}_G + \frac{\partial \mathbf{D}_G}{\partial t} \tag{37}$$

where ρ_G is the density of local rest mass in the local rest frame of the matter, and $\mathbf{j}_G$ is the local rest-mass current density in this frame (in the case of classical matter, $\mathbf{j}_G = \rho_G \mathbf{v}$, where $\mathbf{v}$ is the coordinate three-velocity of the local rest mass; in the quantum case, see Eq. (45)). Here $\mathbf{H}_G$ is the gravitomagnetic field intensity, and $\mathbf{D}_G$ is the gravitodisplacement field. Since the forms of these equations are identical to those of Maxwell's equations, the same boundary conditions follow from them, and therefore the same solutions for electromagnetic problems carry over formally to the gravitational ones. These include the solution for the optimal impedance-matching problem for a thin, dissipative film (see Appendix A).

The constitutive relations (assuming an isotropic medium) analogous to those in Maxwell's theory are

$$\mathbf{D}_G = 4\kappa_{GE}\varepsilon_G \mathbf{g} \tag{38}$$

$$\mathbf{B}_G = \kappa_{GM}\mu_G \mathbf{H}_G \tag{39}$$

$$\mathbf{j}_G = -\sigma_G \mathbf{g} \tag{40}$$

where ε_G is the gravitoelectric permittivity of free space given by Eq. (24), μ_G is the gravitomagnetic permeability of free space given by Eq. (21), κ_{GE} is the gravitoelectric dielectric constant of a medium, κ_{GM} is its gravitomagnetic relative permeability, and σ_G is the gravitational analog of the electrical conductivity of the medium, whose magnitude is inversely proportional to its viscosity. It is natural to choose to define the constitutive relation, Eq. (40), with a minus sign, so that for *dissipative* media, σ_G is always a *positive* quantity. The factor of 4 on the right hand side of Eq. (38) implies that Newton's law of universal gravitation emerges from Einstein's theory of GR in the correspondence principle limit of Newtonian gravity in free space.

The phenomenological parameters κ_{GE}, κ_{GM}, and σ_G must be determined by experiment. Since there exist no negative masses which can give rise to a gravitational analog of the polarization of the medium, we expect that at low frequencies, $\kappa_{GM} \to 1$. However, because of the possibility of large Meissner-like effects such as in superconductors, κ_{GM} need not approach unity at low frequencies, but can approach zero instead. Also, note that κ_{GM} can be spatially inhomogeneous near the surface of a superconductor.

Again, converting the Maxwell-like equations for weak gravity into a wave equation for free space in the standard way, we conclude that the speed of gravitational waves in free space is $c = (\varepsilon_G \mu_G)^{-1/2}$, which is identical in GR to the vacuum speed of light, and that the impedance of free space for gravitational waves is $Z_G = (\mu_G/\varepsilon_G)^{1/2}$, whose numerical value is given in Eq. (22).

It should be stressed here that although the above Maxwell-like equations look formally identical to Maxwell's, there is a basic physical difference between gravity and electricity, which must not be overlooked. In electrostatics, the existence of both signs of charges means that both repulsive and attractive forces are possible, whereas in gravity, only positive signs of masses, and only attractive gravitational forces between masses, are observed. One consequence of this experimental fact is that whereas it is possible to construct Faraday cages that completely screen out electrical forces, and hence electromagnetic radiation fields, it is impossible to construct gravitational analogs of Faraday cages that screen out ordinary gravitational forces, such as Earth's gravity, which are gravitoelectric in nature.

However, the gravitomagnetic force can be either repulsive or attractive in sign, unlike the gravitoelectric force. For example, the gravitomagnetic force between two parallel current-carrying pipes changes sign, when the direction of the current flow is reversed in one of the pipes, according to the Ampere-like law Eq. (37). Hence *both* signs of this kind of gravitational force are possible. One consequence of this is that gravitomagnetic forces *can* cancel out, so that, unlike gravitoelectric fields, gravitomagnetic fields *can* in principle be screened out of the interiors of material bodies. A dramatic example of this is the complete screening out of the Lense-Thirring field by superconductors in a Meissner-like effect, i.e., the complete expulsion of the gravitomagnetic field from the interior of these bodies, which is predicted in the next Section. Therefore the expulsion of gravitational radiation fields by superconductors can also occur, and thus mirrors for this kind of radiation, although counterintuitive, are not impossible.

4 Ginzburg-Landau equation coupled to both electromagnetic and gravitational radiation

A superconductor in the presence of the electromagnetic vector potential $\mathbf{A}(t)$ alone is well described by the Ginzburg-Landau (G-L) equation for the complex order parameter ψ, which in the adiabatic or quasi-static limit is given by [16]

$$\frac{1}{2m_{2eff}}\left(\frac{\hbar}{i}\boldsymbol{\nabla}-e_2\mathbf{A}(t)\right)^2\psi+\beta|\psi|^2\psi=-\alpha\psi. \tag{41}$$

When $\mathbf{A}$ is time-independent, this equation has the same form as the time-independent Schrödinger equation for a particle (i.e., a Cooper pair) with mass m_{2eff} and a charge e_2 with an energy eigenvalue $-\alpha$, except that there is an extra nonlinear term whose coefficient is given by the coefficient β, which arises at a microscopic level from the Coulomb interactions between Cooper pairs [16]. The values of these two phenomenological parameters must be determined by experiment. There are two important length scales associated with the two parameters α and β of this equation, which can be obtained by a dimensional analysis of Eq. (41). The first is the *coherence length*

$$\xi=\sqrt{\frac{\hbar^2}{2m_{2eff}|\alpha|}}\ , \tag{42}$$

which is the length scale on which the condensate charge density $e_2|\psi|^2$ vanishes, as one approaches the surface of the superconductor from its interior. The second is the *London penetration depth*

$$\lambda_L=\sqrt{\frac{\hbar^2}{2m_{2eff}\beta|\psi|^2}}\rightarrow\sqrt{\frac{\varepsilon_0 m_{2eff}c^2}{e_2^2|\psi_0|^2}}\ , \tag{43}$$

which is the length scale on which an externally applied magnetic field $\mathbf{B}(t)=\boldsymbol{\nabla}\times\mathbf{A}(t)$ vanishes due to the Meissner effect, as one penetrates into the interior of the superconductor away from its surface. Here $e_2|\psi_0|^2$ is the condensate charge density deep inside the superconductor, where it approaches a constant.

The G-L equation represents a mean-field theory of the superconductor at the macroscopic level, which can be derived from the underlying microscopic BCS theory [17]. The meaning of the complex order parameter $\psi(x,y,z)$ is that it is the Cooper-pair condensate wavefunction. The G-L theory is being used here because it is more convenient than the BCS theory for calculating the response of the superconductor to electromagnetic, and also to gravitational, radiation.

I propose to generalize the Ginzburg-Landau equation to include gravitational radiation fields arising from the gravitomagnetic vector potential $\mathbf{h}(t)$ by the use of the minimal-coupling rule, Eq. (7), to the following equation:

$$\frac{1}{2m_{2eff}}\left(\frac{\hbar}{i}\boldsymbol{\nabla}-e_2\mathbf{A}(t)-m_2\mathbf{h}(t)\right)^2\psi+\beta|\psi|^2\psi=-\alpha\psi. \tag{44}$$

Again, the ultimate justification for this equation has to come from experiment. With this equation, one can predict what happens at the interface between the superconductor and the vacuum, when both kinds of radiation are impinging on this surface at an arbitrary angle of incidence (see Figure 1). Note that since there are still only two parameters α and β in this equation, there will again be only the same two length scales ξ and λ_L that we had, $before$ adding the gravitational radiation term $\mathbf{h}(t)$. Since there are no other length scales in this problem, we would expect that the gravitational radiation fields (which are strongly coupled to the electromagnetic radiation fields through the coupling Hamiltonian $H_{int} \propto \mathbf{A} \cdot \mathbf{h}$) should vanish on the same length scales as the electromagnetic radiation fields as one penetrates into the interior of the superconductor. Thus we would expect to see a Meissner-like expulsion of the gravitational radiation fields from the superconductor, just like the expulsion of electromagnetic radiation fields.

Both $\mathbf{B}(t)$ and $\mathbf{B}_G(t)$ fields should vanish into the interior of the superconductor, since both $\mathbf{A}(t)$ and $\mathbf{h}(t)$ fields must vanish in the interior. Otherwise, the single-valuedness of ψ would be violated. Suppose that $\mathbf{A}(t)$ did not vanish deep inside the superconducting slab, which is topologically singly connected. Then the nonintegrable phase factor $\exp\left((ie_2/\hbar)\oint \mathbf{A}(t)\cdot d\mathbf{l}\right)$ would also not vanish, which would lead to a violation of the single-valuedness of ψ. Similarly, suppose that $\mathbf{h}(t)$ did not vanish. Then the nonintegrable phase factor $\exp\left((im_2/\hbar)\oint \mathbf{h}(t)\cdot d\mathbf{l}\right)$ would also not vanish, so that again there would be a violation of the single-valuedness of ψ.

There exists much experimental evidence that the single-valuedness of ψ is in fact *not* violated: for example, the quantization of the orbital angular momentum of atoms and molecules in microscopic physics, and the quantization of circulation in superfluid helium, and of flux in superconductors in macroscopic physics. As a special case of the latter when the winding number is zero, the Meissner effect is itself evidence for the validity of the principle of the single-valuedness of ψ.

The $\mathbf{A}(t)$ and $\mathbf{h}(t)$ fields are coupled strongly to each other through the $e_2\mathbf{A}\cdot\mathbf{h}$ interaction Hamiltonian. Since the electromagnetic interaction is very much stronger than the gravitational one, the exponential decay of $\mathbf{A}(t)$ on the scale of the London penetration depth should govern the exponential decay of the $\mathbf{h}(t)$ field. Thus both $\mathbf{A}(t)$ and $\mathbf{h}(t)$ fields decay exponentially with the same length scale λ_L into the interior of the superconductor. This implies that both electromagnetic and gravitational radiation fields will also be expelled from the interior, so that a flat surface of this superconductor should behave like a plane mirror for both electromagnetic and gravitational radiation.

The Cooper-pair current density $\mathbf{j}$, which acts as the source in Ampere's law in both the Maxwell and the Maxwell-like equations, can be obtained in a manner similar to that for the Schrödinger equation

$$\mathbf{j} = \frac{\hbar}{2im_{2eff}}\left(\psi^*\boldsymbol{\nabla}\psi - \psi\boldsymbol{\nabla}\psi^*\right) - \frac{e_2}{m_{2eff}}|\psi|^2\mathbf{A} - \frac{m_2}{m_{2eff}}|\psi|^2\mathbf{h}\ . \tag{45}$$

Note thar $\mathbf{j}$ is nonlinear in ψ, but linear in $\mathbf{A}$ and $\mathbf{h}$. Near the surface of the superconductor, the gradient terms dominate, but far into the interior, the $\mathbf{A}$ and the $\mathbf{h}$ terms dominate. Let us use $\mathbf{j}$ for calculating the sources for both Maxwell's equations for the electromagnetic fields, and also for the Maxwell-like equations for the gravitational fields. The electrical current density is

$$\mathbf{j}_e = e_2\mathbf{j} \tag{46}$$

and the rest-mass current density is

$$\mathbf{j}_G = m_2\mathbf{j}. \tag{47}$$

Also, the electrical free charge density is

$$\rho_e = e_2|\psi|^2 \tag{48}$$

and the rest-mass density is

$$\rho_G = m_2|\psi|^2. \tag{49}$$

I have not yet solved the generalized Ginzburg-Landau equation, Eq. (44), coupled to both the Maxwell and Maxwell-like equations through these currents and densities. These coupled equations are nonlinear in ψ, but are linear in $\mathbf{A}$ and $\mathbf{h}$ for weak radiation fields. However, from dimensional considerations, I can make the following remarks. The electric field $\mathbf{E}(t)$ should vanish exponentially towards the interior of the superconductor on a length scale set by the coherence length ξ, since the charge density $\rho_e = e_2|\psi|^2$ vanishes exponentially on this length scale near the surface of the superconductor. Similarly, the magnetic field $\mathbf{B}(t)$ should vanish exponentially towards the interior of the superconductor, but on a different length scale set by the London penetration depth λ_L. Both fields vanish exponentially, but on different length scales.

At first sight, it would seem that similar considerations would apply to both the gravitational fields $\mathbf{g}(t) \equiv \mathbf{E}_G(t)$ and $\mathbf{B}_G(t)$. However, since there exists only one sign of mass for gravity, the gravitoelectric field $\mathbf{E}_G(t)$ cannot be screened out, as can the electric field $\mathbf{E}(t)$. However, the gravitomagnetic field $\mathbf{B}_G(t)$ *can* be, indeed *must* be, screened out by quantum-mechanical mass currents, in order to preserve the single-valuedness of ψ.

However, the behavior of the superconductor as an efficient *mirror* is no guarantee that it should also be an efficient *transducer* from one type of radiation to the other. For efficient power conversion, a good transducer impedance-matching process from one kind of radiation to the other is also required.

The transducer impedance-matching process should happen naturally in type II superconductors, where the electric field decays more quickly than the magnetic field into the interior of the superconductor, since $\xi < \lambda_L$. For extreme type II superconductors, such as the high-temperature superconductor YBCO, ξ is much less than λ_L by over two orders of magnitude [18]. Therefore, the wave impedance $Z = E/H$ of the electromagnetic plane wave decreases exponentially as a function of z, the distance from the surface into the interior of the superconductor, as

$$Z(z) = \frac{E(z)}{H(z)} = Z_0 \exp(-z/\xi + z/\lambda_L). \tag{50}$$

The gravitational wave impedance Z_G, however, behaves very differently, because of the *absence* of the screening of the gravitoelectric field, so that $E_G(z)$ should be a constant independent of z near the surface, and therefore

$$Z_G(z) = \frac{E_G(z)}{H_G(z)} = Z_G \exp(+z/\lambda_L). \tag{51}$$

Thus the z-dependence of the ratio of the two kinds of wave impedances should obey the exponential-decay law

$$\frac{Z(z)}{Z_G(z)} = \frac{Z_0}{Z_G} \exp(-z/\xi). \tag{52}$$

Let us convert the two impedances Z_0 and Z_G to the same units for the purposes of comparison. To do so, let us express Z_0 in the natural units of the quantum of resistance $R_0 = h/e^2$, where e is the electron charge. Likewise, let us express Z_G in the corresponding natural units of the quantum of dissipation $R_G = h/m^2$, where m is the electron mass. Thus we get the dimensionless ratio

$$\frac{Z(z)/R_0}{Z_G(z)/R_G} = \frac{Z_0/R_0}{Z_G/R_G} \exp(-z/\xi) = \frac{e^2/4\pi\varepsilon_0}{4Gm^2} \exp(-z/\xi). \tag{53}$$

Let us define the "depth of natural impedance-matching" z_0 as the depth where this dimensionless ratio is unity, and thus where natural impedance matching occurs. Thus I find that

$$z_0 = \xi \ln\left(\frac{e^2/4\pi\varepsilon_0}{4Gm^2}\right) \approx 97\xi. \tag{54}$$

This result is a robust one, in the sense that the logarithm is very insensitive to changes in numerical factors of the order of unity in its argument. From this it follows that it is necessary to penetrate into the superconductor a distance of z_0, which is around a hundred coherence lengths ξ (the average value of ξ is around 16 Å in the case of YBCO), for this natural impedance-matching process to occur. When this happens, an efficient transducer impedance-matching process occurs automatically, and we expect the transducer power-conversion efficiency from electromagnetic to gravitational radiation, and vice versa, should be of the order of unity. The London penetration depth is around 2720 Å in the case of YBCO, which is larger than the depth $z_0 \approx 97\xi \simeq 1600$ Å in this material, so that the electromagnetic field energy has not yet decayed much at the natural impedance-matching plane at $z = z_0$, although it is mainly magnetic in character at this depth inside the superconductor.

Of course, the Fresnel-like boundary-value problem for plane waves incident on the surface of the superconductor at arbitrary incidence angles and arbitrary polarizations (see Figure 1) needs to be solved completely before this conclusion can be confirmed. However, based on these crude dimensional and physical arguments, the prospects for a simple, Hertz-like experiment testing these ideas appear to be promising enough that I am presently performing this experiment with Walt Fitelson. The schematic of this experiment is shown in Figure 2. The results of this experiment will be reported elsewhere.

5 Conclusions

In addition to its fundamental interest as an experimental testing ground at the intersection of QM and GR, superconductors can be used as antennas and transducers for an efficient and reciprocal coupling of electromagnetic and gravitational radiation fields. A Hertz-like experiment is presently being performed to test these ideas. If this experiment is successful, superconductors can serve as the basis for practical devices in gravity radio communications, especially in light of the fact that all classical matter, including the Earth, is essentially completely transparent to gravitational radiation. An important follow-up astrophysical experiment would be to observe the cosmic microwave background in *gravitational* radiation, as this would tell us much about the very early Universe.

6 Appendix A: Optimal impedance matching of a gravitational plane wave into a thin, dissipative film

Let a gravitational plane wave given by Eq. (14) be normally incident onto a thin, dissipative (i.e., viscous) fluid film. Let the thickness d of this film be very thin compared to the gravitational analog of the skin depth $(2/\kappa_{GM}\mu_G\sigma_G\omega)^{1/2}$, and to the wavelength λ. The incident fields calculated using Eqs. (32) (here I shall use the notation $\mathbf{E}_G$ instead of $\mathbf{g}$) are

$$\mathbf{E}_G^{(i)} = -\frac{1}{2}(x,-y,0)\omega h_+ \sin(kz-\omega t) \tag{55}$$

$$\mathbf{H}_G^{(i)} = -\frac{1}{2Z_G}(y,x,0)\omega h_+ \sin(kz-\omega t). \tag{56}$$

Let ρ be the amplitude reflection coefficient for the gravitoelectric field; the reflected fields from the film are

$$\mathbf{E}_G^{(r)} = -\rho\frac{1}{2}(x,-y,0)\omega h_+ \sin(kz-\omega t) \tag{57}$$

$$\mathbf{H}_G^{(r)} = +\rho\frac{1}{2Z_G}(y,x,0)\omega h_+ \sin(kz-\omega t). \tag{58}$$

Similarly the transmitted fields on the far side of the film are

$$\mathbf{E}_G^{(t)} = -\tau \frac{1}{2}(x, -y, 0)\omega h_+ \sin(kz - \omega t) \tag{59}$$

$$\mathbf{H}_G^{(t)} = -\tau \frac{1}{2Z_G}(y, x, 0)\omega h_+ \sin(kz - \omega t), \tag{60}$$

where τ is the amplitude tranmission coefficient. The Faraday-like law, Eq. (35), and the Ampere-like law, Eq. (37), when applied to the tangential components of the gravitoelectric and gravitomagnetic fields parallel to two appropriately chosen infinitesimal rectangular loops which straddle the thin film, lead to two boundary conditions which yield the following two algebraic relations:

$$1 + \rho - \tau = 0 \tag{61}$$

$$1 - \rho - \tau = (Z_G \sigma_G d)\,\tau \equiv \zeta\tau \tag{62}$$

where we have used the constitutive relation $\mathbf{j}_G = -\sigma_G \mathbf{E}_G$ (Eq. (40)) to determine the current enclosed by the infinitesimal rectangular loop in the case of the Ampere-like law, and where we have defined the positive, dimensionless quantity $\zeta \equiv Z_G \sigma_G d$. The solutions are

$$\tau = \frac{2}{\zeta + 2} \text{ and } \rho = -\frac{\zeta}{\zeta + 2}. \tag{63}$$

Using the conservation of energy, we can calculate that the absorptivity A, i.e., the fraction of power absorbed from the incident gravitational wave and converted into heat, is

$$A = 1 - |\tau|^2 - |\rho|^2 = \frac{4\zeta}{(\zeta + 2)^2}. \tag{64}$$

To find the condition for maximum absorption, we calculate the derivative $dA/d\zeta$ and set it equal to zero. The unique solution for maximum absorptivity occurs at

$$\zeta = 2, \text{ where } A = \frac{1}{2} \text{ and } |\tau|^2 = \frac{1}{4} \text{ and } |\rho|^2 = \frac{1}{4}. \tag{65}$$

Thus the optimal impedance-matching condition into the thin, dissipative film, i.e., when there exists the maximum rate of conversion of gravitational wave energy into heat, occurs when the dissipation in the fluid film is $Z_G/2$ per square element. At this optimum condition, 50% of the gravitational wave energy will be converted into heat, 25% will be transmitted, and 25% will be reflected. This is true independent of the thickness d of the film, when the film is very thin. This solution is formally analogous to that of the optimal impedance-matching problem of an electromagnetic plane wave into a thin ohmic film [11].

7 Acknowledgments

This paper is an abbreviated version of my lecture and book chapter for the Symposium/Festschrift in honor of Wheeler's 90th birthday. I dedicate this paper to my teacher, John Archibald Wheeler, whose vision helped inspire this paper. I would like to thank Sam Braunstein, Andy Charman, Marv Cohen, Paul Davies, Francis Everitt, Walt Fitelson, Larry Hall, Jandir Hickmann, Jon Magne Leinaas, Bob Laughlin, Colin McCormick, Morgan Mitchell, Joel Moore, Jan Myrheim, Richard Packard, Raul Ramos, Paul Richards, Dan Solli, Charles Townes, Sandy Weinreb, Bill Wootters, and Alex Zettl for their help, and for many stimulating questions and discussions. I would especially like to thank my father-in-law, the late Yi-Fan Chiao, for his financial and moral support of this work.

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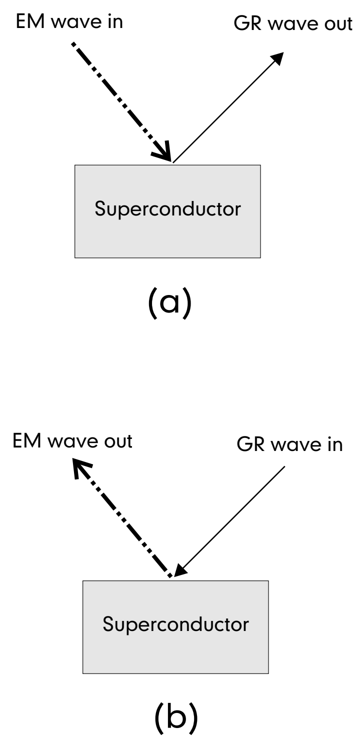

Figure 1: Superconductor as a naturally impedance-matched transducer between electromagnetic (EM) and gravitational (GR) radiation. (a) A quadrupolar EM plane wave is converted upon reflection into a quadrupolar GR plane wave. (b) The reciprocal (or time-reversed) process in which a quadrupolar GR plane wave is converted upon reflection into a quadrupolar EM plane wave. Both EM and GR waves possess the same *quadrupolar* polarization pattern (see Eqs. (13) and (14)). Estimates of the transducer power-conversion efficiency based on Eq. (54) yield efficiencies of the order of unity for extreme type II superconductors such as YBCO.

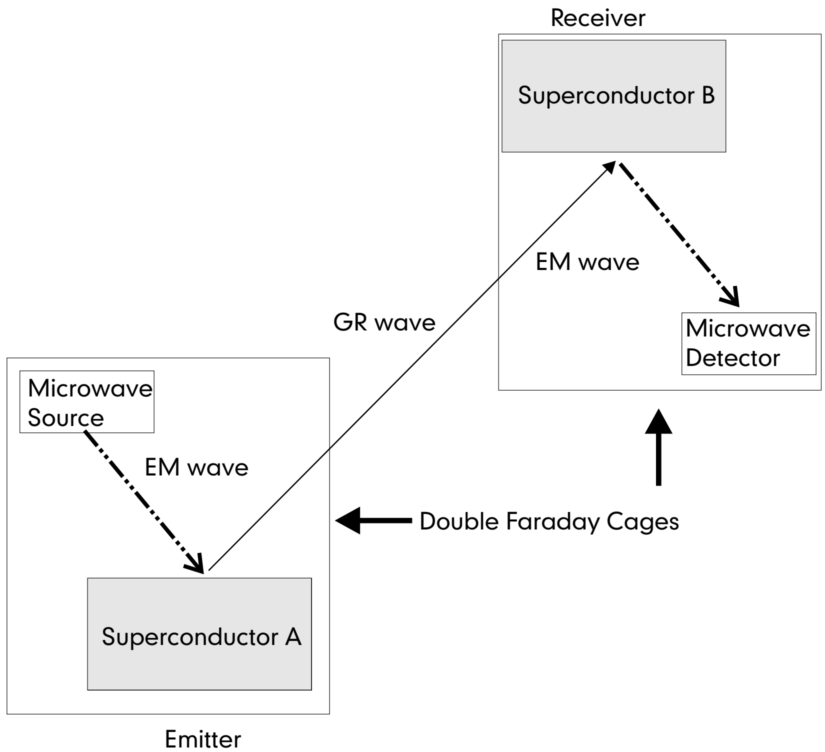

Figure 2: Schematic of a simple, Hertz-like experiment, in which gravitational radiation at 12 GHz is emitted and received using two superconductors. The "Microwave Source" generates quadrupolar electromagnetic radiation at 12 GHz ("EM wave"), which impinges on Superconductor A (a 1 inch diameter piece of YBCO, which is placed inside a dielectric Dewar containing liquid nitrogen), and is converted upon reflection into quadrupolar gravitational radiation ("GR wave"). The GR wave, but not the EM wave, passes through the "Double Faraday Cages," i.e., two doubly-nested normal-metal Faraday cages. In the far field of Superconductor A, Superconductor B (also a 1 inch diameter piece of YBCO inside a dielectric Dewar filled with liquid nitrogen) reconverts upon reflection the quadrupolar GR wave back into a quadrupolar EM wave at 12 GHz, which is then detected by the "Microwave Detector." The GR wave, and hence the signal at the microwave detector, should disappear once either superconductor is warmed up above its transition temperature (90 K), i.e., after the liquid nitrogen boils away in either Dewar.